SIMULTANEOUS MEASUREMENT OF FILM THICKNESS AND WALL SHEAR STRESS IN WAVY FLOW OF NON-NEWTONIAN LIQUIDS

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The film thickness and wall shear stress were measured simultaneously by electrodiffusional and capacitance methods. Experimental data were confronted with the existing theories of gravity flow of non-Newtonian liquids in wavy films with free surface.

Contemporary views[1,2] about dynamics of the periodical two-dimensional gravity film flow with free surface are predominantly based on the integrated equation of continuity

$$\partial_x \int_0^h v_x \, dy + \partial_t h = 0 \tag{1}$$

and the integrated momentum balance in the longitudinal direction

$$\varrho \left[\partial_x \int_0^h v_x^2 \, dy + \partial_t \int_0^h v_x \, dy \right] = \sigma h \, \partial^3_{xxx} h - \tau + \varrho g h \,, \tag{2}$$

where some second order terms are neglected. The film thickness $h = h(x, t)$ and kinematics of the flow, represented by the mean velocity $u = u(x, t)$ and the wall shear rate

$$\gamma = \partial_y v_x(y, x, t)|_{y=0} \,, \tag{3}$$

are dependent parameters, which are related by the balance relations (*1*) and (*2*).

Theories of the wavy film flow have been predominantly verified[1,2] by measurement of film thickness until now. Simultaneous measurements of velocity profiles and film thickness were conducted, for their experimental complexity, only in one known case[2]. The instantaneous shear velocity near the wall can be measured by the electrodiffusional method, which is relatively simple to realize[3,4]. Electrodiffusional experiments have not been treated in velocities[3] or have been interpreted erroneously[5] owing to the neglection of time inertia of the electrodiffusional probe.

In this study a simple theory of the dynamics of an electrodiffusional probe is suggested and experimentally proved. By means of this theory the measured current $I = I(t)$ could be interpreted as the wall shear rate $\gamma = \gamma(t)$ or the corresponding wall shear stress $\tau = \tau(t)$ with an accuracy about 5%. Simultaneous data on $h = h(t)$ and $\tau = \tau(t)$ are confronted with existing theories of the wavy film flow.

Wavy Film Flow of Power-law Liquid

Several authors[6-9] tried to solve the stabilized periodic wavy flow of non-Newtonian liquids by integral methods[1,2]. These procedures were based on the integral balances (*1*), (*2*), on the power-law viscosity model

$$\tau[\gamma] = K[\gamma]^{1/m}, \tag{4}$$

and on the assumption of similarity of velocity profiles

$$v_x(y; x, t) = u(x, t)\frac{2+m}{1+m}\left(1 - \left(1 - \frac{y}{h(x,t)}\right)^{m+1}\right). \tag{5}$$

If the longitudinal momentum on the LHS of Eq. (*2*) is correctly expressed, these assumptions lead to the following set of equations for h, u and γ:

$$\gamma = (m+2)\,u/h, \tag{6}$$

$$\partial_x(uh) + \partial_t h = 0 \tag{7}$$

and

$$\varrho\left[\frac{2(m+2)}{(2m+3)}\partial_x(u^2h) + \partial_t(uh)\right] = \sigma h\,\partial^3_{xxx}h - \tau[\gamma] + \varrho g h. \tag{8}$$

A more detailed treatment would show, that the terms of the order $0((\partial_x h)^{1+n})$, where $n = 1/m = 1$, are neglected in the momentum balance (*8*). From this follows, that it is fully justified to consider the asymptotic solution with the accuracy to the terms of the order $0(h_0\,\partial_x\phi)$

$$h = h_0(1 + \phi(x, t)). \tag{9}$$

The mean film thickness, $h_0 = \langle h \rangle$, and the mean flow rate, $u_0 = q_s/h_0$, are not generally equal to h_s, u_s for the steady non-wavy flow with a wetting density q_s,

$$h_s = ((m+2)\,\varkappa^m q_s)^{1/(m+2)}, \tag{10a}$$

$$u_s = ((m+2)^{-1}\,\varkappa^{-m} q_s^{m+1})^{1/(m+2)}. \tag{10b}$$

However within the framework of the asymptotic solutions, $|\phi| \ll 1$, the relations $h_0 = h_s$, $u_0 = u_s$ and also $\gamma_0 = \gamma_s$, $\tau_0 = \tau_s$, where

$$\gamma_s = (m + 2)\, u_s/h_s = ((m + 2)\, \varkappa^{-2} q_s)^{m/(m+2)}\,, \tag{10c}$$

$$\tau_s = \varrho g h_s = K(\gamma_s)^n = K((m + 2)\, \varkappa^{-2} q_s)^{1/(m+2)} \tag{10d}$$

hold with an error of the order $0(\phi^{1+n})$.

The requirement of constant wave celerity, $c = c_0$, where

$$c = \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_{h=\mathrm{const.}} = -(\partial_t h/\partial_x h)\,, \tag{11}$$

and the requirement of periodicity of the solution lead to simplifications, which render possible to obtain the continuity equation in the integral form

$$(u - c_0)\, h = (u_0 - c_0)\, h_0 \tag{12}$$

and to eliminate the unknown function $u = u(x - c_0 t)$ from the equation of motion (*8*). Periodic solution of the problem can be found in the form

$$\phi(x, t) = \alpha \sin(T)\,, \tag{13}$$

where

$$T = 2\pi(x - Z u_0 t)/\lambda_0 \tag{14}$$

$$Z = c_0/u_0 \approx m + 2 \tag{15}$$

$$\lambda_0 = 2\pi(m(m + 2))\, \varrho u_s^2 (\sigma h_s)^{-1/2} \tag{16}$$

and α is an arbitrary, sufficiently small relative amplitude, $0 < \alpha \ll 1$.

Two alternative expressions for the wall shear rate γ and the corresponding wall shear stress τ follow from the given solution. The first one from the equation of continuity,

$$\gamma + 0(\phi^{1+n}) = (m + 2)\,[u_0 + c(h - h_0)]/h^2 = \gamma_s(1 + Z\phi)/(1 + \phi)^2 \tag{17a, b}$$

$$\tau + 0(\phi^{1+n}) = K\gamma^n = \tau_s[(1 + Z\phi)/(1 + \phi)^2]^n \tag{17c, d}$$

and the second one from the momentum balance,

$$\gamma + 0(\phi^{1+n}) = (h/\varkappa)^m = \gamma_s(1 + \phi)^m \tag{18a, b}$$

$$\tau + 0(\phi^{1+n}) = \varrho g h = \tau_s(1 + \phi)\,. \qquad (18c, d)$$

The expressions according to Eqs (*17*) and (*18*) coincide in the terms of the order $0(\phi)$ inclusively, *i.e.* within the accuracy of the given solution.

For estimation of the amplitude α, the solution containing terms of the order $0(\phi^{1+n})$ must be taken into account even in the case $\phi \to 0$. All these second order solutions lead to results, where z, u_0, λ_0, h_0, α depend on two rheodynamic criteria, Reynolds number

$$\mathrm{Re} = \frac{\varrho u_s^2}{\tau_s} = \frac{1}{g}\left(\frac{q_s^{2m+1}}{(m+2)^3 \varkappa^{3m}}\right)^{1/(m+2)} \qquad (19a)$$

and Weber number

$$\mathrm{We} = \frac{\sigma}{\varrho u_s^2 h_s} = \frac{\sigma}{\varrho}\left(\frac{(m+2)\varkappa^m}{q_s^{2m+3}}\right)^{1/(m+2)}. \qquad (19b)$$

A correct solution of the above mentioned type for non-Newtonian liquid has not been proposed yet. The Shkadov's analyses[10,11] are considered to be correct for the Newtonian liquid. However these analyses predict

$$Z \min(\phi) < -1 \qquad (20)$$

according which the shear rate must be negative in a certain phase of the wave, see Eqs (*17a, b*) and Fig. 1. The only experimental study on velocity profiles[13], reviewed *e.g.* in the work[2], indicates, that γ is always positive and in a qualitative agreement with the prediction based on the momentum balance, see Eq. (*18*).

Dynamics of Electrodiffusional Probe

The electrodiffusional method for scanning of flow kinematics[3,4,14] makes use of the fact, that the electric current on the working electrode

$$I(t) = F_z D \iint_A (-\partial_y c|_{y=0})\,\mathrm{d}A \qquad (21)$$

is fully controled, under convenient conditions, only by convective diffusion of the depolarizer to the surface of the probe. Under conditions of unidirectional flow along the electrode,

$$v_x = \gamma_w(t)\, y\,, \quad v_y = v_z = 0\,, \qquad (22)$$

see Fig. 2, the concentration field of the depolarizer can be obtained by integration

of the transport equation

$$\partial_t c + \gamma_w(t)\, y\, \partial_x c = D\, \partial^2_{yy} c \tag{23}$$

with the boundary conditions expressing the assumption of the limiting current regime on the electrode:

$$c = 0\,; \quad \text{for} \quad x > 0 \quad \text{and} \quad y = 0 \tag{24a}$$

and giving a known concentration of the depolarizer outside the region of the diffusion layer on the electrode

$$c \to c^{\mathrm{B}}\,; \quad \text{for} \quad x \to 0 \quad \text{or} \quad y \to \infty\,. \tag{24b}$$

At a given diffusivity D and concentration c^{B} of the depolarizer in the bulk liquid, the probe response $I(t)$ depends only on $\gamma(t)$. The aim of the theory is to express this dependence quantitatively in such a way, that $\gamma_w(t)$ could be obtained from the known response $I(t)$.

For a steady flow, $\gamma_w(t) = \gamma_s$, exact solutions of the given problem are known for different shapes of the electrode[14]. Especially for the electrode of the circular cross

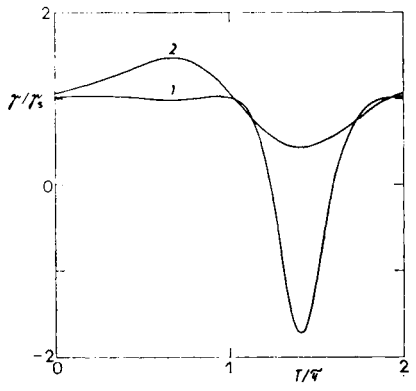

FIG. 1

Theoretical prediction of flow kinematics close the wall for Newtonian liquid, $m = 1$. *1* γ according to continuity equation, Eq. (*17b*), *2* γ according to momentum balance, Eq. (*18b*), ϕ is calculated according to Kapitsa[12]: $\phi = \alpha \sin T + 0{\cdot}28\alpha^2 \cos 2T - 0{\cdot}5\alpha^2 [(Z - 1)(Z - 1{\cdot}2)]^{-3/2}$. $\mathrm{We}^{1/2}\mathrm{Re}^{-1} \sin 2T$; $\alpha = 0{\cdot}46$, $Z = 2{\cdot}4$ and $\mathrm{We}^{1/2}\mathrm{Re}^{-1} = 5{\cdot}0$

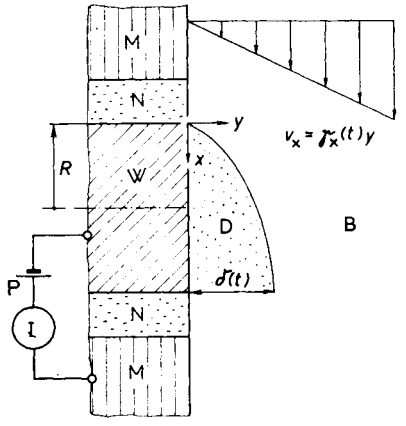

FIG. 2

Electrodiffusional probe. W working electrode of circular cross section, N electrical insulation, M auxiliary electrode (vertical stainless steel wetted plate), P source of constant potential, I current registration, D diffusional layer on surface of working electrode, B bulk of liquid, $\delta(t)$ boundary layer thickness at rear edge of working electrode, R electrode radius. For $\gamma_x(t)\, y$ read $\gamma_w(t)\, y$

section with the radius R holds[3,4,14]

$$k(t) = k_s = 0{\cdot}6866 D^{2/3} R^{-1/3} \gamma_s^{1/3}\,, \tag{25}$$

where

$$k(t) = I(t)/(F_z c^B \pi R^2)\,. \tag{26}$$

Exact solutions are known also for the so-called potentiostatic transient process[14], which is equivalent to the response $I(t)$ of the stepwise change of the shear rate from infinity to a constant value

$$\gamma_w(t) = \begin{cases} \infty\,; & t < 0 \\ \gamma_s\,; & t > 0\,. \end{cases} \tag{27}$$

The exact solution of the transient process for a circular electrode can be expressed by the following relation[14] with accuracy better than 0·02%

$$k(t) = \begin{cases} k_s \theta^{-1/2}(1 + 0{\cdot}1535\theta^{3/2} - 0{\cdot}0042\theta^{9/2})\,; & \theta < 2{\cdot}582 \\ k_s\,; & \theta > 2{\cdot}582\,, \end{cases} \tag{28a, b}$$

where

$$\theta = t/t_s\,, \tag{29}$$

$$t_s = 0{\cdot}6752 D^{-1/3} R^{2/3} \gamma_s^{-2/3}. \tag{30}$$

Relatively rough approximative solutions for more general dependence of $\gamma_w(t)$ can be obtained under the assumption, that $\gamma_w(t)$ does not differ much from its mean value, $\gamma_w(t) = \gamma_s + \tilde{\gamma}(t)$, $|\tilde{\gamma}(t)| \ll \gamma_s$, in other words, that the problem can be linearized. Examples of such approach have been shown and applied in the study of turbulence spectra in liquids flowing through channels[3,4]. Such approach is inconvenient for time dependent flows, where the condition $|\tilde{\gamma}(t)| \ll \gamma_s$ is not fulfilled. Among them belong also the film flows with a free surface.

In this paper dynamic response of the probe on the change of shear rate is simulated by the following simple approximative model. In the transport equation (*23*), the concentration field is assumed to be a similarity function of three variables

$$c(y; x, t) \approx c^B \left(1 - F\left(\frac{y}{\delta(t)}\left(\frac{2R}{x}\right)^{1/3}\right)\right), \tag{31}$$

where $F(\zeta)$ is an appropriately chosen decreasing function, which is normalized by the conditions $F(0) = 1$, $F(\infty) = 0$, $F'(0) = -1$. The substitution of Eq. (*31*) into Eq. (*23*), integration over $y \in (0; \infty)$ and then over $(x, z) \in A$ lead to the result in

the form

$$\gamma_w(t) = b_1 \left(k^3 + b_2 \frac{dk}{dt} \right), \tag{32}$$

where b_1, b_2 are free constants, which depend for a given fluid on the function $F(\zeta)$ and on the shape of the electrode. They can be estimated either by an appropriate calibration procedure of the electrode in the given liquid, or from the known exact solutions.

From the known solutions for the steady process, Eq. (*25*) and the start-up period $t \to 0$ of the transient process, Eq. (*28a*), the following approximative theoretical expression for a circular electrode can be obtained

$$\gamma_w(t) = \frac{Rk^3}{(0{\cdot}6866)^3 D^2} \left(1 + \frac{2D}{\pi k^3} \frac{dk}{dt} \right). \tag{33}$$

For the stepwise change of the shear rate from γ_0 to γ_s, the differential equation (*33*) has the simple analytical solution

$$\frac{t}{t_s} = \int_{k_s/k_0}^{k_s/k} \frac{2s \, ds}{1 - s^3}, \tag{34}$$

where $k_0 = 0{\cdot}6866 D^{2/3} R^{-1/3} \gamma_0^{1/3}$. Specifically for $\gamma_0 = \infty$, *i.e.* for the stepwise change according to Eq. (*27*), Eq. (*34*) reduces to the result, which should be identical with the exact solution according to Eqs (*28a, b*). From Fig. 3 follows, that even in this most unfavourable case, the instantaneous values of the current calculated according to the approximative theory do not differ more than 3% from the exact theory.

Symbol $\gamma_E(t)$ will be used for the quasisteady interpretation of the measured signal,

$$\gamma_E = \gamma_E(t) = \frac{R \, k^3(t)}{(0{\cdot}6866)^3 D^2}. \tag{35}$$

By reversed substitution of Eq. (*35*) into Eq. (*33*) we have obtained the expression, which is already convenient for practical treatment of the current signal $I(t)$ into the shear rate

$$\gamma_w(t) = \gamma_E \left(1 + 0{\cdot}45 D^{-1/3} R^{2/3} \gamma_E^{-5/3} \frac{d\gamma_E}{dt} \right). \tag{36}$$

In diagnostic praxis[3,4] the described type of the probe was used above all for measurement of the wall shear stress τ. For inelastic fluids, which do not exhibit slip effects on the wall, the instantaneous values of $\gamma(t)$ and $\tau(t)$ are mutually bound by

a viscosity function. Specifically for the power-law model of viscosity function, see Eq. (4), the expression for τ under steady conditions follows from Eq. (*35*)

$$\tau_E = \tau_E(t) = K\left(\frac{R\,k^3(t)}{(0{\cdot}6866)^3\,D^2}\right)^{1/m} = (\chi\, I^3(t))^{1/m}, \tag{37}$$

where

$$\chi = \frac{K^m}{10{\cdot}03 D^2 R^5 (c^B F_z)^3} \tag{38}$$

and the corresponding correction on the dynamics of convective diffusion in the region close to the working electrode from Eq. (*36*)

$$\tau_w(t) = \tau_E\left(1 + \beta\tau_E^{-(1+2m/3)}\frac{\mathrm{d}\tau_E}{\mathrm{d}t}\right)^{1/m}, \tag{39}$$

where

$$\beta = 0{\cdot}45m\, D^{-1/3}\, K^{2m/3}\, R^{2/3} \tag{40}$$

is constant for a given electrode and liquid in a large interval of values τ_w.

Calibration of Electrodiffusional Shear Stress Probe

It follows from the dynamic theory of the electrodiffusional shear stress probe, see foregoing paragraph, that the calibration parameters χ, β depend on both electrochemical and rheological properties of the fluid. Water solutions of CMC (Na-carboxymethylcellulose Lovosa 20) and PAA (polyacrylamid Separan AP 30) containing 25 mol m^{-3} of the working depolarizer $Fe(CN)_6^{3-}$, the same amount of the auxiliary depolarizer $Fe(CN)_6^{4-}$, 57 mol m^{-3} of inert ions SO_4^{2-} and the corresponding amount of common K^{1+} ions were used. The other relevant characteristics of the solutions are summarized in Table I.

The parameters K, m of the power-law viscosity function were determined from the experimental data obtained on a Brookfield RVT and Rheotest II viscometers in the range of shear rate $3-300\ s^{-1}$. Temperature of the solutions was held at $295{\cdot}2 \pm 0{\cdot}2$ K.

The working electrode of the probe was formed by the front surface of a platinum wire with the radius $R = 95$ μm, which was flush polished with the surrounding insulation N and the vertical ground plate M made of stainless steel, see Fig. 2. The plate wetted with the film of a test fluid served simultaneously as the auxiliary electrode.

The calibrations of the electrodiffusional probe were carried out in two arrangements, which are shown in Fig. 4.

Under the conditions of non-wavy flow, *i.e.* at a sufficiently small flow rate q_s and the small distance of the probe from the distributor, $d = 0{\cdot}17$ m, it was possible to calibrate the probe in the film with a free surface. The film thickness h_C was measured by the capacity probe simultaneously with the limiting diffusion current and τ_C was evaluated from the relation

$$\tau_C = \varrho g h_C . \tag{41}$$

The arrangement shown in Fig. 4*b* was used for stabilization of the flow at a higher flow rate. By means of spacers, which do not appear in the figure, the organic glas plate G was fixed parallel to the plate M in the distance $2h_s$. The flow rate was so adjusted, that a stable dynamic meniscus

TABLE I

Solution parameters and calibration constants of the probe (temperature 295 K)

Polymer	CMC	PAA
Polymer conc., % b.wt	3·0	0·6
ϱ, kg m^{-3}	1 030	1 020
K, Pa $s^{1/m}$	0·42	0·12
m_{rheom}	1·25	1·41
m_{calib}	1·20	1·33
χ, $Pa^m A^{-3} m^6$	$47{\cdot}9 \cdot 10^{16}$	$8{\cdot}63 \cdot 10^{16}$
D_{calib}, $m^2 s^{-1}$	$0{\cdot}823 \cdot 10^{-9}$	$0{\cdot}796 \cdot 10^{-9}$
D_{trans}, $m^2 s^{-1}$	—	$0{\cdot}762 \cdot 10^{-9}$
β, $Pa^{2m/3} s$	0·60	0·21

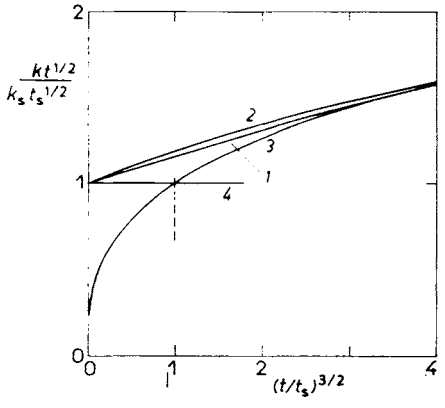

FIG. 3

Reaction of electrodiffusional probe on step change of shear rate. *1* exact theory, Eqs (*28a, b*), *2* approximative theory, Eq. (*34*), *3* steady asymptote k = const. for $t \to \infty$, *4* penetration asymptote $k \cdot t^{1/2}$ = const. for $t \to 0$

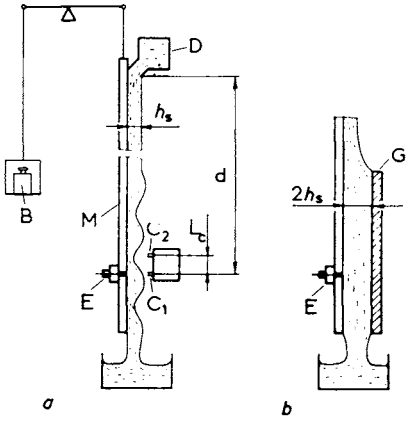

FIG. 4

Experimental arrangement for calibration of electrodiffusional probe and measurement. *a* vertical film with free surface, *b* vertical channel, E electrodiffusional probe, C_1, C_2 capacitance probes, D distributor, B balance for hold-up measurement, G organic glass plate, M vertical stainless steel plate with length 0·6 m and width 0·15 m

has formed at the entrance of the channel. After correction on capillary effects in dynamic menisci and inertia effects in the entrance and exit of the channel, which were smaller than 5%, the wall shear stress was evaluated from Eq. (*10d*).

Primary calibration data for both solutions taken under steady conditions are shown in Fig. 5. Numerical treatment of these data led to values of the constant $m = m_{calib}$, see Table I. The estimation of $D = D_{calib}$, given also in Table I, was calculated by means of the theoretical expression (*38*) with the given values $K = K_{rheom}$, $R = 95 \cdot 10^{-6}$ m and χ. An alternative estimation $D = D_{trans}$ can be determined from the data on the transient current characteristics measured after the stepwise change of the polarization potential from the equilibrium to the constant value $\eta = -0{\cdot}6$ V. The later value guarantees the regime of limiting current in the given system. The exact and approximative theory, Eqs (*28*) and (*33*), result in the common equation

$$It^{1/2} = a + bt^{3/2} , \tag{42}$$

where b depends on γ_s or τ_s and for the parameter $a = \lim_{t \to 0} (It^{1/2})$ the following relation holds

$$D_{trans} = \pi a / (F_z c^B \pi R^2)^2 . \tag{43}$$

The data on the transient characteristics in PAA solution are shown in Fig. 6 for two thicknesses of the channel, $2h_s = 1{\cdot}0$ and $2{\cdot}0$ mm. Both sets of the data result in the common value $a = (1{\cdot}06 \pm 0{\cdot}005)\ 10^{-6}\ As^{1/2}$. Excellent agreement of D_{trans} with D_{calib} is documented in Table I. Values of β characterize the dynamics of the probe under time dependent flow, $\tau = \tau(t)$. They are calculated according to the definition (*40*) for the given $D = D_{calib}$ and appear in Table I.

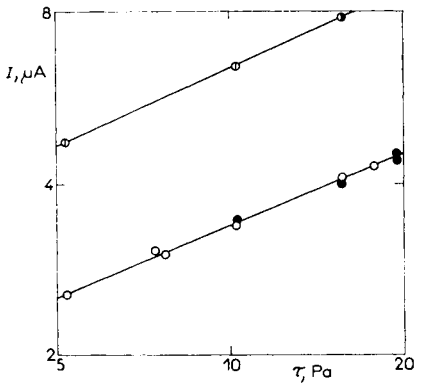

FIG. 5

Calibration data under steady conditions. CMC: ○ measurement with free surface, ● measurement in channel; PAA: ⊖ measurement with free surface, ◑ measurement in channel. From straight lines corresponding to theory (*37*) calibration constants χ and m_{calib} (see Table I) were calculated

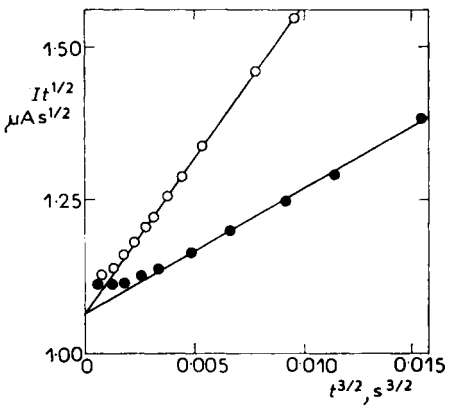

FIG. 6

Transient current characteristics of electrodiffusional probe for PAA solution; ○ $\tau_s = 10$ Pa, ● $\tau_s = 5$ Pa. Straight lines correspond to asymptotes according to Eq. (*42*)

Dynamics of the probe is fully characterized by pair of the calibration constants χ, β. Eqs (*37*) and (*39*) make possible to caluclate $\tau(t)$ from the measured signal $I(t)$.

Calibration and Use of Capacitance Probes

The principle of the capacitance method was in detail described in the paper[15]. With the aim to measure simultaneously film thickness and wave celerity, a couple of the probes was used in this study. The diameter of the probes was 3 mm and the lower capacitance probe, C_1, was placed against the electrodiffusional probe, see Fig. 4*a*. As the dielectric constant of water solutions is negligibly small in comparison to the dielectric constant of air, the probe C_i can be calibrated by adjustment of the gap, Δh [m], between the front surface of the probe and the dry plate M and by reading the voltage, U_i [V], which is the measure of the capacity of this condenser. The calibration of the probe gave the empirical expression

$$\Delta h = 10^{-3}\left[\left(\frac{1}{64} + \frac{10}{(1 + U_1)^2 - 0{\cdot}0952}\right)^{1/2} - 0{\cdot}225\right]. \qquad (44)$$

With regard to the above mentioned values of dielectric constant, the value Δh was taken as the gap between the probe and the liquid film surface during the experiments.

The wave celerity c was calculated from the expression

$$c = L_c/t_c\,, \qquad (45)$$

where $L_c = 2 \cdot 10^{-2}$ m is the distance between the capacitance probes and t_c [s] is the time in which the crest of a wave passed the probes.

Simultaneous Measurement of Film Thickness and Wall Shear Stress

The runs were carried out with two above mentioned polymer solutions in the arrangement shown in Fig. 4*a*. Measurement of flow rate, pumping and liquid distribution into regular film, hold-up and capacitance film thickness measurement were in detail described in preceedings papers[15,16].

Flow rate from the distributor D was held constant. The probes E, C_1 were located at the distance 0·49 m downstream the distributor exit. Flow in the examined region could be characterized as partially stabilized wavy flow[2]. Amplitude and wave celerity was constant at least at the distance of three wave lengths from the measuring point.

Primary signals of the capacitance and electrodiffusional probe were normalized on voltage signals 0—10 V, sampled with frequency 100 Hz, converted to 8 bit data and stored in the memory of a transient recorder. In the treatment of the data, the film thickness $h(t)$ was expressed as the shear stress $\tau_C(t)$ according to Eq. (*41*) and the current $I(t)$ as the equivalent quasisteady shear stress $\tau_E(t)$ according to the calibration relation (*37*) where the calibration constants from Table I were used.

Typical results on simultaneous scanning of film thickness and wall shear stress appear in Fig. 7. The measuring conditions are characterized in Table II.

DISCUSSION

Simultaneous records of film thickness and current signal of electrodiffusional probe have been already taken[3,5], but they have not been converted into comparable informations on hydrodynamics of the wavy flow. For this aim, the signal of electro-

TABLE II
Parameters of film flow

Record	7*a*	7*b*	7*c*	7*d*
Solution	CMC	CMC	CMC	PAA
q_s, $m^2\ s^{-1}$	$0{\cdot}87 \cdot 10^{-4}$	$1{\cdot}42 \cdot 10^{-4}$	$1{\cdot}88 \cdot 10^{-4}$	$0{\cdot}33 \cdot 10^{-4}$
τ_s, Pa	16·7	19·4	21·2	6·4
$\varrho g\langle h_C\rangle$, Pa	16·4	19·5	21·5	5·2
$\varrho g h_B$, Pa	17·2	20·9	22·6	5·8
Re	0·165	0·280	0·379	0·42
We	15·0	6·55	4·07	40·0
u_s, ms^{-1}	$5{\cdot}21 \cdot 10^{-2}$	$7{\cdot}32 \cdot 10^{-2}$	$8{\cdot}87 \cdot 10^{-2}$	$5{\cdot}16 \cdot 10^{-2}$
c, ms^{-1}	0·136	0·211	0·227	0·246
Z	2·61	2·88	2·56	4·75
λ_0, m	$2{\cdot}02 \cdot 10^{-2}$	$1{\cdot}55 \cdot 10^{-2}$	$1{\cdot}33 \cdot 10^{-2}$	$1{\cdot}16 \cdot 10^{-2}$
λ_{obs}, m	$4{\cdot}17 \cdot 10^{-2}$	$5{\cdot}65 \cdot 10^{-2}$	$5{\cdot}06 \cdot 10^{-2}$	$11{\cdot}4 \cdot 10^{-2}$
t_{obs}, s	0·307	0·267	0·223	0·466
t_s, s	0·069	0·062	0·057	0·036

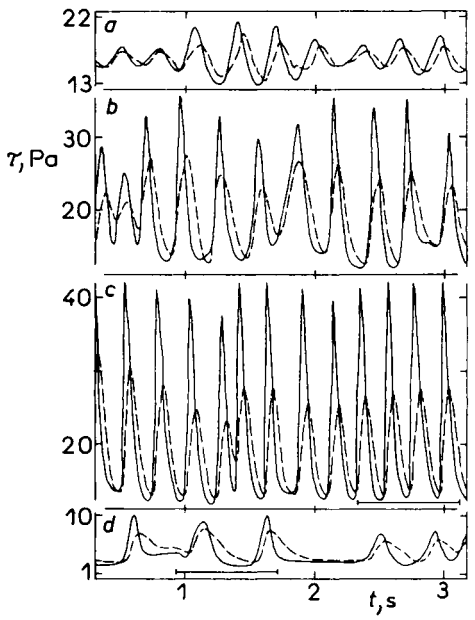

FIG. 7

Equivalent shear stress. Solid line — signal from capacitance probe converted to τ_C according to Eq. (*41*); dashed line — signal from electrodiffusional probe converted to τ_E according to quasisteady theory, Eq. (*37*); *a*, *b*, *c*, and *d* — records specificated in Table II. Parts of records marked by abscissas in Figs 7*c*, *d* are shown in Fig. 8 in slow motion scale

diffusional probe must be corrected on inertia effect, which is caused by finite velocity of concentration changes of depolarizer in the diffusional layer on the working electrode. Parts of the records from Figs 7*c*, *d* are shown in Figs 8*a*, *b* in slow motion coordinate. In the later figures wall shear stress $\tau_w(t)$ appears as well, which was obtained by treatment of $\tau_E(t)$ according to Eq. (*39*).

For the inelastic solution of CMC the data $\tau_C(t)$ and $\tau_w(t)$ can be considered as identical within the limits of measurement accuracy. This is in agreement with the theoretical results on wavy flow dynamics, according which Eq. (*18*), *i.e.* $\tau = \varrho g h$, is fulfilled with accuracy of the order $0(h/\lambda_s)^{1+n}$ for long waves, *i.e.* $h \ll \lambda_s$. Dis-

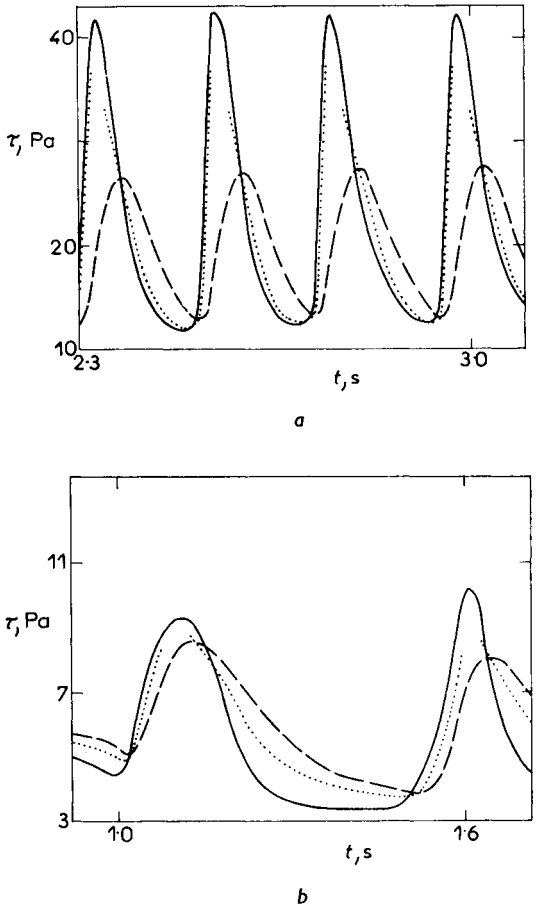

FIG. 8

Correction of signal of electrodiffusional probe on time inertia. Solid and dashed lines have the same meaning as in Fig. 7, dotted line stands for corrected τ_w, see Eq. (*39*). *a* CMC, part of record from Fig. 7*c*, *b* PAA, part of record from Fig. 7*d*

crepancy between experimental results and theoretical estimation (*17*), which admits even negative values of τ, can be explained by the fact, that the assumptions of theoretical analyses are not fulfilled at real wavy flows of liquids with a higher consistency. The following assumptions are especially under discussion:

(i) The wavy flow has achieved stabilized periodic state, $v_x = v_x(y; x - c_0 t) = v_x(y; x + \lambda_0 - c_0 t)$.

(ii) The cross pressure changes of the order $0(\partial_x h)^{1+n}$ and the corresponding cross flow can be neglectes.

(iii) The velocity field can be approximated by the similarity profile, Eq. (*5*).

It would be interesting to develop an alternative approach to the study of wavy film flow dynamics, which would take account of the experimentally supported hypothesis $\tau = \varrho g h + 0(\partial_x h)^2$, under conditions when the assumptions (i), (ii) are not fulfilled.

Signals $\tau_C(t)$ and $I(t)$ scanned in this paper are very similar to those in the studies[3,5]. For inelastic liquids we have shown, that asymmetry of the signal $I(t)$ and his time lag behind the signal $\tau_C(t)$ can be explained by the time inertia of electrodiffusional probe.

For the highly elastic PAA solution $\tau_C(t)$ and $\tau_E(t)$ have different shapes and the correction on the inertia of the probe does not help much. There are two probable causes of this discrepancy.

Electrodiffusional probe reacts on the intensity of convective transport near the working electrode. The convection is characterized by the shear rate, $\gamma(t)$, not by the shear stress. This does not matter in time independent (inelastic, nontixotropic liquids) where the instantaneous values $\gamma(t)$, $\tau(t)$ are directly related by a viscosity function. In elastic liquids with relaxation times which are comparable with the fluctuation period of $\gamma(t)$ or $\tau(t)$, the relaxation process can substantially affect the relation between $\gamma(t)$ and $\tau(t)$. The signal $\tau_w(t)$ in Fig. 8*b* relaxes to the value $\tau_C(t)$, which is for $t \in (1{\cdot}3; 1{\cdot}5)$ s nearly constant. The corresponding estimate of the relaxation time equals about 0·1 s and is in agreement with the data on similar PAA solutions[17].

The other possible cause of the discrepancy between $\tau_C(t)$ and $\tau_w(t)$ in the PAA solution can be a different dynamics of the wavy film flow of highly elastic liquids.

Both mentioned pecularities of the wavy film flow of elastic liquids, as well as the questions of quantitative interpretation of electrodiffusional signal will be the subject of the next investigations.

LIST OF SYMBOLS

a	parameter of penetration asymptote, Eq. (*42*), A $s^{1/2}$
A	surface of working electrode, m^2
c	concentration of depolarizer, mol m^{-3}
c^B	concentration of depolarizer in bulk, mol m^{-3}

c local wave celerity, Eq. (*11*), m s^{-1}
c_0 constant group wave celerity, m s^{-1}
d distance of measurement point from distributor, Fig. 4, m
D diffusivity of depolarizer, m^2 s^{-1}
D_{calib} value of D estimated by calibration under steady conditions
D_{trans} value of D measured under potentiostatic transient process
F_z Faraday constant, charge corresponding to electrode reaction of 1 mole of depolarizer, A s mol^{-1}
g gravitational acceleration, m s^{-2}
h instantaneous local thickness of two-dimensional film, m
h_s mean value h of non-wavy flow, Eq. (*10a*), m
h_0 theoretical mean value h of wavy flow, m
h_C local film thickness calculated from capacitance probe, m
$\langle h_C \rangle$ mean value of h_C
h_B film thickness estimated from hold-up
I current signal of electrodiffusional probe, A
k current signal converted to transfer coefficient, Eq. (*26*), m s^{-1}
k_s value of k under steady conditions, Eq. (*25*), m s^{-1}
K consistency coefficient of fluid, Pa s^n
L_c distance between capacitance probes C_1, C_2; m
m reciprocal flow index, Eq. (*4*)
$n = 1/m$ flow index
m_{rheom} value m from viscometric data, Eq. (*4*)
m_{calib} value m from electrodiffusional data, Eq. (*37*)
p pressure, Pa
q instantaneous local flow rate, m^2 s^{-1}
q_s mean flow rate, m^2 s^{-1}
R radius of circular working electrode, m
Re Reynolds number, Eq. (*19a*)
t time, s
t_{obs} time between passing of crests of two subsequent waves, s
t_s characteristic time of electrodiffusional probe, Eq. (*30*), s
$u = q/h$ mean flow velocity , m s^{-1}
u_0 time mean of u, m s^{-1}
u_s value of u for non-wavy flow, Eq. (*10b*), m s^{-1}
U_i voltage signals of capacitance probe, $i = 1, 2$ V
v_x, v_y longitudinal and perpendicular component of velocity
x, y longitudinal and perpendicular coordinate, Fig. 2
We Weissenberg number, Eq. (*19b*)
Z dimensionless group celerity
α relative wave amplitude, Eq. (*13*)
β calibration constant of electrodiffusional probe, Eq. (*39*), Tab. I, $Pa^{2m/3}$ s
γ viscometric shear rate, shear rate close to wall, s^{-1}
γ_s γ value of non-wavy flow, Eq. (*10c*), s^{-1}
γ_0 γ mean value of wavy flow, s^{-1}
γ_E electrodiffusional signal converted to γ under steady conditions, Eq. (*35*), s^{-1}
γ_w electrodiffusional signal converted to γ under dynamic conditions, Eq. (*36*), s^{-1}
δ thickness of diffusional layer at rear edge of working electrode, Eq. (*31*), m
η working potential of electrodiffusional probe, V

θ normalized time variable, Eq. (*29*)
$\varkappa = K/\varrho g$
ϕ relative wave thickness, Eq. (*9*)
λ_0 theoretical wave lenght, Eq. (*16*), m
λ_{obs} observed wave length, m
χ calibration constant of electrodiffusional probe, Eq. (*37*), Table I, $Pa^m A^{-3} m^6$
ϱ liquid density, kg m^{-3}
σ surface tension, Pa m
τ viscometric shear stress, instantaneous local wall stress, Pa
τ_s value τ in non-wavy flow, Pa
τ_C signal of capacitance probe converted to τ, Eq. (*41*), Pa
τ_E signal of electrodiffusional probe converted to τ under steady conditions, Eq. (*37*), Pa
τ_w signal of electrodiffusional probe converted to τ under dynamic conditions, Eq. (*39*)

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